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EXPECTATIONS AND INTERTEMPORAL SEPARABILITY IN AN EMPIRICAL MODEL OF CONSUMPTION AND INVESTMENT UNDER UNCERTAINTY

by Philippe J. Deschamps*

An intertemporal model of consumption and investment under uncertainty is formulated, and compared with the existing literature; it is argued that an assumption of myopia is necessary for its empirical applicability. It is estimated by maximum likelihood with quarterly British data. A specification search for a satisfactory form of expectations is made, and the estimated model is compared with a static demand system. Strong intertemporal separability is formulated as a nested hypothesis, and strongly rejected by a likelihood ratio test.

1. Introduction[1]

Most existing intertemporal models of individual behavior can be classified according to four criteria. The first one specifies whether the economic agent faces a two-period horizon, or maximizes his objective over the life cycle. The second one specifies the agent's expectations, which can be deterministic or stochastic. In the latter case, one may either make specific assumptions on the density of the uncertain variables, or assume that the expectations are "rational", i.e. assume only that the expectations of the uncertain variables exist, conditional on all the information currently available. The third criterion determines whether the model is of consumption only, portfolio only, or consumption and portfolio (we define a portfolio model as one where investment is optimally diversified into several financial assets, and where some future asset returns are uncertain). The fourth criterion is the intertemporal separability, or nonseparability, of the utility function.

Table 1 classifies 18 well-known contributions into 8 model classes according to the four preceding criteria. The first class is the pioneering Tintner-Hicks extension of the static theory of demand, later reformulated for continuous time in the first part of Lluch and Morishima (1973, p.169ff). In the second class, the model of Fama (1970, 1976) handles both consumption and investment decisions under uncertainty over the life cycle and does not assume intertemporal separability. Consumption is treated as aggregate, an assumption considered by Epstein (1975) as restrictive. The third class encompasses the mean-variance models of portfolio selection. In this group, the empirical models of Parkin (1970) and Saito (1977) are probably the most specific, since they rely on an explicit utility function and on explicit distributions of asset returns. As is well-known, however, this has a price in terms of flexibility. In the fourth group, the second part of Lluch and Morishima (1973, p.177ff) extends the Tintner-Hicks analysis to the case where price (but not income) expectations are stochastic. The models in groups 5 and 6 are two-period only. Morishima (1952) and Allingham and Morishima (1973) rationalize the agent's decisions by introducing a "future standard of living" into his (separable) utility function. The model in Epstein (1975) gives a disaggregate treatment of consumption, but does not explicitly consider portfolio decisions; however, it handles income uncertainty

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as well as price uncertainty. In the seventh group, we have the empirical paper by Bronsard and Salvas-Bronsard (1986), which appears to depend on deterministic expectations as we will see. Lastly, we have the rational expectations models of Hall (1978), Browning et al. (1985) and Attfield and Browning (1985). The popularity of rational expectations, and its lack of restrictiveness when compared to other specifications, gives high significance to the models in this class. However, the Hall rational expectations formulation whereby the marginal utility of consumption follows a random walk with trend appears to depend crucially on intertemporal separability, a fact acknowledged by Browning et al. (1985, p.510).

TABLE 1: MODEL CHARACTERISTICS

		Life-cycle	Uncertainty	Consumption	Portfolio	Intertemporal Separability
1	Tintner (1938) Hicks (1946) Lluch-Morishima (1973, 169ff.)	Yes	No	Yes	No	Yes
2	Fama (1970)	Yes	Yes	Yes	Yes	No
3	Tobin (1958) Markowitz (1959) Bierwag-Grove (1968) Parkin (1970) Morishima (1973) Saito (1977)	No	Yes	No	Yes	Yes
4	Lluch-Morishima (1973, 177ff.)	Yes	Yes	Yes	No	No
5	Morishima (1952) Allingham-Morishima (1973)	No	Yes	Yes	Yes	Yes
6	Epstein (1975)	No	Yes	Yes	No	No
7	Bronsard et al. (1986)	Yes	No	Yes	No	No
8	Hall (1978) Browning et al. (1985) Attfield-Browning (1985)	Yes	Yes	Yes	No	Yes

The theoretical model in Section 2 of this paper was largely inspired by the contributions mentioned in Table 1, and we do not claim that its general specification is original. We do claim, however, that it is disaggregate, and can include equations for any number of commodities and any number of assets; that it is flexible, and does not rely on an explicit utility function or probability distribution of asset returns; that it does not assume intertemporal separability, but includes it as a nested, testable case; that it is formulated as a life-cycle model, and can therefore be compared with the existing multiperiod literature; and that it encompasses model classes 1 through 7 in Table 1. However, our aim to formulate a model with empirical content *and* a test of intertemporal separability later forces us to specify behavior that is myopic, in a sense that will be made precise and that will be rationalized in the life cycle context. It also effectively prohibits us from using the Hall (1978) rational expectations formulation, so that our model does not encompass class 8 in Table 1. Our defense is that we make a specification search for the form of expectations that "works best", and present tolerably good estimates when that form is chosen.

An outline of the paper follows. In Section 2, we present our life-cycle optimization model, and substantiate our claim that it encompasses groups 1 to 7 in Table 1. We also formulate and justify our maintained restriction of myopic behavior. In Section 3, we derive a differential demand system under the general restrictions of economic theory. The comparative statics of the problem are studied, and our results are compared with some of the literature. Section 4 gives a detailed presentation of the specification search for the form of the expectations; it also presents the empirical estimation of the consumption and investment model under adding-up and symmetry, using British quarterly data on seven consumption goods and on savings. The results are compared with an ordinary, static, demand system. Section 5 incorporates the additional restriction of strong intertemporal separability (SIS). The new model is shown to be quadrilinear in the parameters. Section 6 presents the estimation of the model under SIS. Section 7 concludes.

Section 2. Optimal consumption and investment decisions

Following a long tradition, we assume that a representative individual has a strictly concave lifetime utility function $u(x_0, \ldots, x_t, \ldots, x_T)$, where x_t is an $n \times 1$ vector of consumption goods. The utility function relevant for the decision problem in period t is $u_t(x_t, \ldots, x_T) \equiv u(x_0, \ldots, x_T)$. The temporary budget constraint for a period $t < T$ is:

$$p_t' x_t + \imath_m' a_{t+1} = w_t + (\imath_m + r_t)' a_t \tag{1}$$

where p_t is the $n \times 1$ vector of commodity prices; w_t is non-capital income; r_t is an $m \times 1$ vector of interest rates or yields; a_t is the $m \times 1$ vector of financial asset holdings at the beginning of period t; and $\imath_m$ is an $m \times 1$ vector of ones (which can be replaced by a vector of transactions costs). Death occurs at the beginning of period T.

At time $t < T$, the budget constraint for the future is a discounted sum of temporary budget constraints, obtained as:

$$\sum_{\tau=t+1}^{T} \beta_{\tau-t}(p_\tau' x_\tau) + \sum_{\tau=t+1}^{T} \beta_{\tau-t}(\imath_m' a_{\tau+1}) = \sum_{\tau=t+1}^{T} \beta_{\tau-t}(w_\tau + \rho_\tau' a_\tau) \tag{2}$$

where $\beta_{\tau-t}$ is a discount function with the normalization $\beta_1 = 1$, and where $\rho_\tau' \equiv (\imath_m + r_\tau)'$.

We now introduce uncertainty, and assume the existence of a predictive density:

$$f_t(p_{t+1},\ldots,p_T,w_{t+1},\ldots,w_T,r_{t+1},\ldots,r_T)$$

conditional on all the information available at the beginning of period t. This density is characterized by a vector π_t containing the parameters of f_t; the elements of π_t may be functions of current and lagged prices, incomes, and interest rates. It will also be helpful to define:

$$X_{t+1}=\begin{pmatrix} x_{t+1}\\ \vdots \\ x_T\end{pmatrix} \quad , \quad A_{t+1}=\begin{pmatrix} a_{t+1}\\ \vdots \\ a_T\end{pmatrix} \quad , \quad P_{t+1}=\begin{pmatrix} \beta_1 p_{t+1}\\ \vdots \\ \beta_{T-t} p_T\end{pmatrix}.$$

As shown by Fama (1970, 1976), the problem of maximizing expected *lifetime* utility $Eu(x_0,\ldots,x_T)$ subject to T temporary constraints of the form (1) can be broken up in a sequence of temporary optimization problems. In our context, these problems are:

$$\max_{x_t,a_{t+1}} \quad E_t u_t(x_t, X^*_{t+1}), \quad \text{subject to:}$$

$$p'_t x_t + \imath'_m a_{t+1} = w_t + \rho'_t a_t \tag{3}$$

$$P'_{t+1} X^*_{t+1} = w_{t+1} + \rho'_{t+1} a_{t+1} + I^*_{t+2} \tag{4}$$

where:

$$\begin{aligned} I^*_{t+2} &= \sum_{\tau=t+2}^{T} \beta_{\tau-t}(w_\tau + \rho'_\tau a^*_\tau) - \sum_{\tau=t+2}^{T} \beta_{\tau-t-1}(\imath'_m a^*_\tau) \quad \text{for } t < T-1 \\ &= 0 \quad \text{for } t = T-1, \end{aligned}$$

where E_t denotes expectation under the density f_t, and where $X^*_{t+1}, a^*_{t+2},\ldots,a^*_T$ are the optimal future consumption and asset holding *plans*. Equations (3) and (4) are simply restatements of (1) and (2).

As noted by Lluch and Morishima (1973) and Epstein (1975), the temporary problem may be solved in two steps. We first define the (stochastic) direct-indirect utility function as the maximum of u_t subject to (4):

$$v_t(x_t, P_{t+1}, w_{t+1} + \rho'_{t+1}a_{t+1} + I^*_{t+2}) = \max_{X_{t+1}}\{u_t(x_t, X_{t+1}) \mid P'_{t+1}X_{t+1} = w_{t+1} + \rho'_{t+1}a_{t+1} + I^*_{t+2}\} \tag{5}$$

and write the conditional expectation of v_t as:

$$E_t v_t(x_t, P_{t+1}, w_{t+1} + \rho'_{t+1}a_{t+1} + I^*_{t+2}) = F_t(x_t, a_{t+1}, \pi_t, A^*_{t+2}) \tag{6}$$

where, as will be recalled, π_t contains the parameters of the density f_t, and where $A^*_{t+2} = (a^*_{t+2},\ldots,a^*_T)$ (an empty vector if $t = T-1$).

The second step determines the optimal current decision as the solution of:

$$(P) \qquad \max_{x_t,a_{t+1}} F_t(x_t, a_{t+1}, \pi_t, A^*_{t+2}), \quad \text{subject to} \quad p'_t x_t + \imath'_m a_{t+1} = w_t + \rho'_t a_t.$$

This solution is written as:

$$\begin{pmatrix} x_t \\ a_{t+1} \end{pmatrix} = \Phi_t(p_t, w_t + \rho_t' a_t, \pi_t, A_{t+2}^*). \tag{7}$$

Note that F_t and Φ_t are *functionals* of the optimal future asset holding plan A_{t+2}^*; loosely speaking, Φ_t depends on the *parameters* of the optimal future policy, and not on any unobserved future variables.

Model (7) can now be compared with the literature in Table 1. Our temporary budget constraint (1) is the discrete-time and multi-asset counterpart of Equation (2) in Lluch and Morishima (1973). Similarly, combining Equation (2) of Fama (1970) and the temporary budget constraint in his model yields our equation (1) for the special case where $p_t' x_t \equiv c_t$ and $w_t = 0$. The mean-variance models in class 3 of Table 1 may be rationalized as follows: the individual maximizes $E_t U(W_{t+1})$ subject to $\imath_m' a_{t+1} = W_t$, where $U(W_{t+1})$ is a utility function of future wealth $W_{t+1} = (\imath_m + r_{t+1})' a_{t+1}$. The yield vector r_{t+1} is stochastic, with a two-parametric density. In this case the expected utility $E_t U(W_{t+1})$ depends on a_{t+1} and on the expectations, variances, and covariances of the asset returns. For a two-period horizon and in the absence of consumption goods, $E_t U(W_{t+1})$ will be recognized as our expected direct-indirect utility function (6), whereas $\imath_m' a_{t+1} = W_t$ is the counterpart of (1) when there are no consumption goods.

In the absence of assets, and for a specific discount function β_{T-t}, the life-cycle budget constraint in Lluch and Morishima (1973, eq. (29)) will be recognized as the sum of our equations (1) and (2). Price expectations take the form of a discrete finite distribution in their article. We may assimilate $w_{t+1} + \rho_{t+1}' a_{t+1} + I_{t+2}^*$ in our equation (5) to the "future standard of living" ξ in Allingham and Morishima (1973); their model then becomes equivalent to ours under a two-period horizon and in the absence of price uncertainty. The variable indirect utility function in Epstein (1975, eq. 2.1) is obviously the same as our equation (5); the constraint in his model is obtained by summing our equations (1) and (2) for $t = 0$ and $T = 1$. Finally, a comparison with Bronsard and Salvas-Bronsard (1986) is interesting, since it illustrates the difficulties involved when one introduces uncertainty. The temporary budget constraint in their article is the same as our equation (1) when $m = 1$, as can be seen by letting $\gamma_t = (1 + r_t)^{-1}$ in their equation (6). But in the absence of uncertainty, the derived utility function reduces to our equation (5) rather than our equation (6): it is a functional of expected income minus expected investment, rather than a functional of the holding plans for *every* asset. This is precisely the specification of the temporary utility function in Bronsard and Salvas-Bronsard (1986, eq. 9), which appears to be nonstochastic.

Equation (7) immediately highlights the problems involved in an empirical formulation of the preceding model. Whereas the optimal plan X_{t+1}^* can be formulated in period t as the solution of the maximization problem in (5), and hence can be made implicit in Equation (7), the same does not hold for $A_{t+2}^* = (a_{t+2}^*, \ldots, a_T^*)$. It is apparent from (7) that a_{T-k}^* depends on A_{T-k+1}^*. Hence the plans a_{T-k}^* must be obtained by backward recursion from T to $T - k$.

An empirical counterpart of (7) can be obtained, however, if we assume that $\beta_2 = \cdots = \beta_{T-t} = 0$ and that, suboptimally, $\imath_m' a_{t+2}^* = \imath_m' a_{t+1}$. In this case $I_{t+2}^* = -\imath_m' a_{t+2}^*$, and Equation (4) becomes $p_{t+1}' x_{t+1}^* = w_{t+1} + r_{t+1}' a_{t+1}$, so that A_{t+2}^* no longer appears in (7). This is one possible rationalization of myopia in a life-cycle context: the (subjective) higher-order discount rates and expected future investment are zero, so that the individual behaves as a two-period maximizer. Zero expected future investment is also assumed by Bronsard and Salvas-Bronsard (1986), as can be seen from their equation (6). Our approach is unusual in that we impose myopic behavior via restrictions on the future budget constraint (4), rather than on intertemporal preferences; we choose to follow it because of the difficulty of suitably restricting preferences, especially when intertemporal separability is not assumed.

Section 3. A differential demand system

For the reasons stated in the preceding section, we assume that the discount function in Equation (2) satisfies $\beta_{\tau-t} = 0$ for $\tau - t \geq 2$, and that expected future investment is zero. For reasons of data availability, we also reformulate Problem (P) in terms of investments $(a_{t+1} - a_t)$ rather than asset holdings. The reformulated problem is:

$$(P') \qquad \max_{x_t,(a_{t+1}-a_t)} F(x_t, (a_{t+1} - a_t), \pi_t, t) \quad \text{subject to} \quad p_t' x_t + \imath_m'(a_{t+1} - a_t) = w_t + r_t' a_t,$$

and Equation (7) can be written as:

$$y_t \equiv \begin{pmatrix} x_t \\ a_{t+1} - a_t \end{pmatrix} = Y(p_t, w_t + r_t' a_t, \pi_t, t). \tag{8}$$

A general specification for the expectations parameters π_t is given by:

$$\pi_t = \Pi(p_t, w_t, r_t, \gamma_t, t) \tag{9}$$

where γ_t includes all the information that is relevant for predicting the uncertain variables, apart from current prices p_t, current non-capital income w_t, and current interest rates r_t; typically, the elements of γ_t will include any number of past observations (p_τ, w_τ, r_τ).[2] Combining (8) and (9) yields:

$$y_t = \bar{Y}(p_t, w_t, r_t, a_t, \gamma_t, t). \tag{10}$$

It is shown in the Appendix that the Slutsky equations for commodities and assets are:

$$\bar{Y}_p = K - Y_w x_t' + Y_\pi \Pi_p \tag{11}$$

$$\bar{Y}_w = Y_w + Y_\pi \Pi_w \tag{12}$$

$$\bar{Y}_r = Y_w a_t' + Y_\pi \Pi_r \tag{13}$$

$$\bar{Y}_a = Y_w r_t' \tag{14}$$

$$\bar{Y}_\gamma = Y_\pi \Pi_\gamma \tag{15}$$

$$\bar{Y}_t = Y_t + Y_\pi \Pi_t \tag{16}$$

where the generic expression F_z indicates a matrix with $\partial F_i / \partial z_j$ in the i-th row and j-th column and where, for simplicity, time indices have been omitted in the matrix derivatives. Note that, from (8), $Y_w = Y_{w+r'a}$.

A brief discussion of Equations (11) to (16) will now be made. In view of Equation (9), the objective F of Problem (P') is a function of the prices p_t; it is well-known in such a case that the matrix $\bar{Y}_p$ of uncompensated price derivatives is the sum of a Slutsky matrix (K), an Engel matrix $(-Y_w x_t')$, and a Veblen matrix $(Y_\pi \Pi_p)$, as in Equation (11); see Allingham and Morishima (1973). Note, however, that K is a rectangular $(m+n) \times n$ matrix in this context, since the last m "prices" are all equal to one; the derivation in the Appendix shows that its upper $n \times n$ block must be symmetric and negative definite. In view of (8), (9) and $Y_w = Y_{w+r'a}$, Equations (12) to (16) follow from the chain rule.

No further comparison between our results and the Slutsky equations obtained by previous authors (e.g. Bierwag and Grove (1968); Morishima (1973), Epstein (1975)) can be made without imposing more structure on our expectations function (9). It should be noted at this stage, however, that the matrix Y_π can be decomposed further into a sum of two matrices, which are separately identifiable under strong intertemporal separability. The discussion of this topic will be postponed until Section 5.

The differential system approach of Theil (1975) and Barten (1969) conveniently leads to an operational counterpart of Equation (10). The latter reads in differential form as:

$$dy_t = \bar{Y}_p dp_t + \bar{Y}_w dw_t + \bar{Y}_r dr_t + \bar{Y}_a da_t + \bar{Y}_\gamma d\gamma_t + \bar{Y}_t dt$$

or, upon substituting Equations (11) to (16) and rearranging terms:

$$dy_t = Y_t dt + K dp_t + Y_w(-x_t' dp_t + dw_t + a_t' dr_t + r_t' da_t) + Y_\pi(\Pi_p dp_t + \Pi_w dw_t + \Pi_r dr_t + \Pi_\gamma d\gamma_t + \Pi_t dt). \quad (17)$$

Upon noting that Equation (1) and the definition $y_t' = (x_t' \quad (a_{t+1}' - a_t'))$ imply:

$$(p_t' \quad \imath_m') dy_t = -x_t' dp_t + dw_t + a_t' dr_t + r_t' da_t$$

and that the bracket in the last term of (17) is simply the differential $d\pi_t$ of Equation (9), we obtain:

$$dy_t = Y_t dt + K dp_t + Y_w(v_t' dy_t) + Y_\pi d\pi_t \quad (18)$$

with $v_t' = (p_t' \quad \imath_m')$.

Equation (18) is simply the generalization of the model in Barten (1969) when the utility function depends on "state variables" π_t and t. The neo-classical treatment of this problem is well-known (see, e.g. Phlips (1974, pp. 180–183)), and a similar demand system can be found in Bronsard and Salvas-Bronsard (1986). The reader may then wonder why the detailed analysis of this section was necessary. Essentially, it shows that the demand system (18) can be characterized by a Slutsky local structure *even if π_t depends on prices and income*, provided that Equation (9) can be independently estimated (so that "observations" on $d\pi_t$ are available, enabling the identification of Y_π). This fact explains an important difference between our model and that of Bronsard and Salvas-Bronsard. Their price coefficient matrix is not equal to K, but to a composite ($K + Y_\pi \Pi_p$ in our notation); our model differs from theirs in that we choose to impose the general restrictions of economic theory on K rather than on $K + Y_\pi \Pi_p$ in order not to restrict the permissible form of the expectations function (9).

Upon defining $\mu_t = v_t' y_t$, multiplying the i-th equation in (18) by v_{it}/μ_t and adding a disturbance term u_{it}, we obtain the Rotterdam parameterization of (18):[3]

$$z_{it} = a_i + \sum_{j=1}^{n} S_{ij} \Delta \log p_{jt} + b_i(\sum_{j=1}^{n+m} z_{jt}) + \sum_{j=1}^{s} S_{ij}^* \Delta \pi_{jt} + u_{it} \tag{19}$$

where $z_{it} = v_{it}\Delta y_{it}/\mu_t$; $a_i = v_{it}(Y_t)_i/\mu_t$; $S_{ij} = v_{it}K_{ij}p_{jt}/\mu_t$; $b_i = v_{it}(Y_w)_i$; $S_{ij}^* = v_{it}(Y_\pi)_{ij}/\mu_t$; and s is the dimension of π_t. For a justification of the assumption that the coefficients a_i, S_{ij}, b_i, and S_{ij}^* in (19) are constant, and for an interpretation of the error term u_{it}, see Mountain (1988). We will denote by a, S, b, S^* the matrices containing the coefficients of (19).

When Equation (19) is estimated from quarterly data, the first differences Δx_t should be defined as $x_t - x_{t-4}$ for the sake of comparability with estimates based on annual data. An additional empirical justification of this definition will be given in the next section.

Equation (19) must be estimated under the adding-up restrictions:[4]

$$\iota_{n+m}'(a \quad S \quad b \quad S^*) = (0 \quad O_{1\times n} \quad 1 \quad O_{1\times s})$$

and under a symmetry restriction on the upper $n \times n$ block of S (it is obvious from Equation (1) that no homogeneity restriction exists in this model). The data on $\Delta\pi_{jt}$ must be generated by making specific assumptions on the function Π in (9), a problem that will be addressed in the following section.

Section 4. Estimation under adding-up and symmetry

Ideally, the data for the preceding model should include observations on commodity demands and asset *holdings*. Barrett, Gray and Parkin (1975) publish a sample of 37 quarterly observations on holdings for 12 groups of financial assets; unfortunately, the list of assets in their article did not appear to be sufficiently exhaustive for a reasonable definition of portfolio income. Indeed, the demand for stocks, for instance, is subject to very large short-run speculative fluctuations that must find a counterpart in variations of substitute assets (such as real estate); if the latter are not included in the analysis, those speculative fluctuations will appear as spurious income variations, and investment will be highly correlated with income. The estimated marginal propensity to spend would then be nearly zero. We therefore estimated the model with quarterly British data on a single composite asset, defined as savings (total personal disposable income minus consumption) and with seven consumption goods: food, drink and tobacco, housing, clothing, fuel and light, durables (excluding vehicles) and miscellaneous goods and services (including transport and vehicles).

Quarterly data on consumer demands and disposable income from 1955 (quarter 1) to 1982 (quarter 2) were taken from the *Economic Trends Annual Supplement*, 1983 edition. Population data were taken from the *Monthly Digest of Statistics* (MDS), July 1969, 1983, and 1984. Price indices were also taken from MDS.

We took as a representative interest rate the gross flat yield on 2.5% consols, available in MDS before May 1979 and in *Financial Statistics* for later periods.

Regarding expectations, we first note that P_{t+2} and I^*_{t+2} no longer appear in Equation (5) under our maintained restriction of myopia, so that f_t can be taken as a predictive density on $(p_{t+1}, w_{t+1}, r_{t+1})$ only, with parameters π_t. On the basis of the multiplicative central limit theorem, there are strong a priori grounds for assuming f_t to be lognormal (see Feldstein (1969)). So π_t could have as many as $(n+m+1)(n+m+4)/2$ elements, consisting of the parameters of a joint normal density on $(\log p_{t+1}, \log w_{t+1}, \log r_{t+1})$. Clearly such a large number of expectations variables would entail multicollinearity and degrees of freedom problems. A feasible alternative is to assume that the second-order moments can be omitted from the list of expectations variables in the demand system (19). This specifies logarithmic point expectations, but only as an approximation; we do not assume that expectations are deterministic. Also, an unfortunate consequence of the lack of suitable data on asset holdings is the impossibility of constructing expectations on non-capital income w_{t+1}. We therefore could not include the predictor $\log \hat{w}_{t+1}$ in the explanatory variables of (19). This assumes, in effect, that $\log \hat{w}_{t+1}$ is the sum of a trend and a constant (which may be seasonal).

To summarize, our vector of expectations variables takes the form:

$$\pi_t = \begin{pmatrix} \log \hat{p}_{1,t+1} \\ \vdots \\ \log \hat{p}_{n,t+1} \\ \log \hat{r}_{t+1} \end{pmatrix} \tag{20}$$

where $\hat{p}_{i,t+1}$ and $\hat{r}_{t+1}$ are predictors of the future prices and interest rate. Three basic forms of prediction were tried. Static expectations sets $\hat{p}_{i,t+1} = p_{i,t-j}$ and $\hat{r}_{t+1} = r_{t-k}$, j and k being chosen by sensitivity analysis. It should be noted that in the context of our differential logarithmic specifications (19) and (20), static price expectations do no imply the equality of past and expected prices, but rather that the expected inflation rates are constant multiples of the past inflation rates. Perfect foresight sets $\hat{p}_{i,t+1} = p_{i,t+1}$ and $\hat{r}_{t+1} = r_{t+1}$. Finally, autoregressive expectations predict the future prices and interest rate from a trend $t+1$, seasonal dummies $d_{j,t+1}$, and four lagged dependent variables, as follows:

$$\begin{aligned} \log \hat{p}_{i,t+1} &= \hat{a}_i(t+1) + \sum_{j=1}^{4} \hat{b}_{ij} d_{j,t+1} + \sum_{j=1}^{4} \hat{c}_{ij} \log p_{i,t-j+1} \qquad (i = 1, \ldots, 7) \\ \log \hat{r}_{t+1} &= \hat{a}_8(t+1) + \sum_{j=1}^{4} \hat{b}_{8j} d_{j,t+1} + \sum_{j=1}^{4} \hat{c}_{8j} \log r_{t-j+1} \end{aligned} \tag{21}$$

where the $\hat{a}_i$, $\hat{b}_{ij}$, and $\hat{c}_{ij}$ are "seemingly unrelated" estimates. The seemingly unrelated model (21) used in predicting (p_{t+1}, r_{t+1}) is estimated for the sample period going from 1948 (quarter 2) to time t. Table 2 presents the results of two such regressions: the first one for the one quarter ahead prediction of the variables in 1955 (quarter 2), based on 28 previous quarters (24 observations); the second one for the one quarter ahead prediction of the variables in 1982 (quarter 3), based on 137 previous quarters (133 observations).

TABLE 2: SAMPLE AUTOREGRESSIONS (EQUATION (21))

Prediction of prices and yield for Quarter 2, 1955

	$\hat{a}_i$	$\hat{b}_{i1}$	$\hat{b}_{i2}$	$\hat{b}_{i3}$	$\hat{b}_{i4}$	$\hat{c}_{i1}$	$\hat{c}_{i2}$	$\hat{c}_{i3}$	$\hat{c}_{i4}$
Food	.0042	-.0052	.0122	-.0118	-.0120	1.1906	-.2349	-.1290	-.0350
	.0023	.0185	.0145	.0153	.0164	.1870	.2910	.3056	.1989
Drink/Tob.	.0007	-.0218	-.0251	-.0188	-.0199	1.0026	-.2137	-.2343	.1083
	.0002	.0054	.0048	.0054	.0054	.1499	.2111	.2102	.1363
Housing	.0021	-.0157	-.0017	-.0174	-.0171	.8828	-.0116	-.2129	.1302
	.0005	.0055	.0048	.0046	.0052	.1649	.2177	.2155	.1515
Clothing	.0005	.0230	.0209	.0118	.0172	1.6160	-.4346	-.7208	.4414
	.0007	.0068	.0062	.0064	.0064	.1663	.3477	.3445	.1643
Fuel/Light	.0044	.0011	-.0250	-.0267	-.0088	.9126	-.0570	.1993	-.3276
	.0016	.0114	.0102	.0123	.0129	.1829	.2753	.2723	.1673
Durables	.0007	.0185	.0023	.0088	.0041	1.7379	-1.1420	.3917	-.1101
	.0006	.0079	.0071	.0073	.0078	.1432	.2845	.2852	.1466
M.Goods/S.	.0015	-.0058	.0001	-.0066	-.0032	1.6080	-.7970	.2178	-.1575
	.0007	.0062	.0054	.0054	.0058	.1788	.3557	.3572	.1776
Yield	.0015	-.8842	-.9200	-.9021	-.9233	1.2607	-.6532	.3844	-.2627
	.0014	.3292	.3272	.3270	.3291	.1703	.2973	.3060	.1914

Prediction of prices and yield for Quarter 3, 1982

	$\hat{a}_i$	$\hat{b}_{i1}$	$\hat{b}_{i2}$	$\hat{b}_{i3}$	$\hat{b}_{i4}$	$\hat{c}_{i1}$	$\hat{c}_{i2}$	$\hat{c}_{i3}$	$\hat{c}_{i4}$
Food	.0001	.0079	.0186	-.0124	.0004	1.1635	-.1188	.2652	-.3137
	.0001	.0045	.0043	.0044	.0046	.0807	.1317	.1316	.0816
Drink/Tob.	.0002	-.0084	.0018	-.0075	-.0116	1.2272	-.2896	.1559	-.0932
	.0001	.0061	.0059	.0060	.0060	.0733	.1126	.1125	.0746
Housing	.0001	.0023	.0239	-.0071	-.0011	1.0258	.0757	-.2937	.2054
	.0001	.0054	.0047	.0052	.0053	.0729	.1070	.1088	.0763
Clothing	.0001	-.0010	.0014	-.0015	.0001	1.5537	-.4338	-.2136	.0871
	.0000	.0021	.0021	.0021	.0021	.0723	.1416	.1414	.0717
Fuel/Light	.0001	.0012	-.0048	.0018	.0154	1.4559	-.8329	.7039	-.3233
	.0002	.0054	.0054	.0055	.0053	.0727	.1231	.1232	.0755
Durables	.0002	-.0012	-.0018	-.0014	.0000	1.4384	-.4096	-.1206	.0875
	.0001	.0031	.0030	.0030	.0031	.0654	.1229	.1230	.0667
M.Goods/S.	.0002	.0005	.0053	-.0017	-.0015	1.2240	-.0989	.0446	-.1732
	.0001	.0030	.0029	.0030	.0030	.0663	.1151	.1156	.0686
Yield	.0011	-.3549	-.3432	-.3481	-.3536	1.2122	-.2651	-.1253	.0744
	.0004	.1140	.1140	.1137	.1138	.0851	.1349	.1357	.0860

Figures show estimated coefficients and estimated asymptotic standard errors.

In our search for the list of expectations variables π_{it} to be included in Equation (19), the following strategy was adopted. We first estimated Equation (19) with 27 candidate expectations variables: static (lagged 3 periods), autoregressive, and perfect foresight expectations on the 7 prices, totaling $3 \times 7 = 21$ regressors; and static (lagged 0, 1, 2, and 3 periods), autoregressive, and perfect foresight expectations on the interest rate, making up the remaining six variables. The rationale behind lagging prices 3 periods is that most prices exhibit strong seasonal variations; this is not the case for the interest rate series. We then estimated nine nested submodels that selectively exclude some of the expectations candidates, and chose the specification with the highest likelihood value.

The nine specifications, and corresponding loglikelihoods, are given in the last nine rows of Table 3, where the column headings p_{t+1}, r_{t+1} denote perfect foresight; $\hat{p}_{t+1}$, $\hat{r}_{t+1}$ denote autoregressive expectations; and p_{t-j}, r_{t-k} denote static expectations with the appropriate lag. A cross under the heading indicates that the corresponding variables were *included* in the demand system. A test of each restricted model versus the full model (corresponding to the first row in the table) can be made with the likelihood ratio test statistic, corrected for small samples (Anderson (1958), pp. 207–210):

$$U = -2\frac{M}{N}\log\lambda, \quad \text{with } M = N - q - \frac{1}{2}(p - q_1 + 1) \tag{22}$$

where N is the number of observations (106 in our case); λ is the likelihood ratio; q is the number of regressors in the *unrestricted* system (39 in our case with seasonal dummies); p is the number of equations in the incomplete demand system ($p = n + m - 1 = 7$ in our case); and q_1 is the number of regressors that are excluded under the null hypothesis.

TABLE 3: SPECIFICATION SEARCH FOR LIST OF EXPECTATIONS VARIABLES

p_{t+1}	$\hat{p}_{t+1}$	p_{t-3}	r_{t+1}	$\hat{r}_{t+1}$	r_t	r_{t-1}	r_{t-2}	r_{t-3}	$\log L$	U	$P(\chi^2 \le U)$
X	X	X	X	X	X	X	X	X	3531.997		
X			X	X	X	X	X	X	3437.010	125.455	0.9678
	X		X	X	X	X	X	X	3429.999	134.714	0.9918
		X	X	X	X	X	X	X	3447.400	111.732	0.8378
		X	X						3409.617	167.407	0.9768
		X		X					3402.727	176.832	0.9934
		X			X				3406.500	171.670	0.9866
		X				X			3411.497	164.835	0.9682
		X					X		3409.998	166.885	0.9752
		X						X	3407.384	170.461	0.9843

The eleventh column in Table 3 gives the nine values of U corresponding to each submodel. This statistic has an approximate Chi-square distribution with pq_1 degrees of freedom under the null hypothesis;

the relevant probabilities $P(\chi^2_{pq_1} \leq U)$ are given in the last column. It is seen that among the three specifications of price expectations that were tried, only static expectations are not rejected at the 5% significance level, since $P(\chi^2_{98} \leq 111.732) = 0.8378 < 0.95$.

We then proceeded, on this assumption of static price expectations, to search for the best specification for the interest rate. This is seen to be static expectations with a one-period lag (r_{t-1}), giving a loglikelihood of 3411.497. Furthermore, at a significance level of 1%, this null hypothesis in row 8 of Table 3 is not rejected against the 27 candidate alternative (row 1 of Table 3), since $P(\chi^2_{133} \leq 164.835) = 0.9682 < 0.99$.

The models in Table 3 were all estimated with seasonal dummies. These, however, were found to be quite insignificant: the statistic U in (22) was equal to 7.93, which is below the *first percentile* (8.90) of the Chi-square distribution with $7 \times 3 = 21$ degrees of freedom. Clearly approximating differentials by $\Delta x_t = x_t - x_{t-4}$ completely deseasonalizes the data; furthermore, experiments with the ordinary Rotterdam model indicated that it is the only first-differencing scheme that preserves the comparability with estimates based on annual data. Seasonal dummies were therefore excluded from the final specification.

Incidentally, price expectations are jointly very significant: a test of their omission from the final specification yielded a test statistic of $U = 84.7$, which is well above the 99-th percentile (74.9) of the Chi-square distribution with $7 \times 7 = 49$ degrees of freedom.

Table 4 presents Model (19) estimated under adding-up only; Table 5 presents the estimates under adding-up and symmetry. All the diagonal elements of the Slutsky matrix are significant except for clothing and fuel and light; all the marginal propensities to spend and save are significant except for housing and fuel and light; and all the significant coefficients have the expected sign. We therefore consider our estimates satisfactory. The magnitude of the marginal propensity to save (.60) can be explained by the definition of disposable income, which includes money and provisions for tax payments (savings thus also includes these).

Regarding expectations, it should be noted that the signs of the coefficients in S^* are not a priori determinate. Both signs can be heuristically justified; on the one hand, an expected price increase may lead to interperiod substitution, raising the current demand; on the other hand, it may well reduce the current demand through an intertemporal income effect. In Table 4, we see that the coefficients of the lagged interest rate and of the last expected price are quite significant in the equation for savings. More surprising is the strong significance of the expected prices of housing and clothing in the equation for housing. The Durbin-Watson statistic for this equation is 1.10, indicating possible serial correlation. This equation was therefore reestimated by the Cochrane-Orcutt method. The expected price of clothing lost some of its significance, with a new coefficient of .042 and an estimated standard error of .023. However, the own (housing) price *and* expected price coefficients remained virtually unchanged and strongly significant; and the marginal propensity to spend remained insignificant. In summary, there was no considerable difference between the Cochrane-Orcutt and OLS estimates for this equation.

TABLE 4: THE MODEL UNDER ADDING-UP

	Cons.	p_1	p_2	p_3	p_4	p_5	p_6	p_7	Inc.
Food	-0.0005	-0.0829	-0.0108	-0.0059	0.0504	-0.0108	0.0057	0.0713	0.0435
	0.0008	0.0136	0.0102	0.0095	0.0258	0.0111	0.0205	0.0211	0.0116
Drink/Tob.	0.0012	0.0314	-0.0713	-0.0080	0.0106	-0.0118	-0.0139	0.0542	0.0419
	0.0007	0.0122	0.0092	0.0086	0.0233	0.0100	0.0185	0.0191	0.0105
Housing	0.0046	0.0127	0.0051	-0.0792	0.0197	-0.0101	-0.0103	0.0279	-0.0061
	0.0007	0.0119	0.0089	0.0083	0.0226	0.0097	0.0179	0.0185	0.0102
Clothing	0.0010	-0.0067	-0.0007	-0.0032	-0.0243	0.0017	0.0076	0.0184	0.0564
	0.0005	0.0093	0.0070	0.0065	0.0177	0.0076	0.0140	0.0145	0.0080
Fuel/Light	0.0006	-0.0067	0.0060	0.0058	0.0099	0.0032	-0.0047	-0.0211	0.0050
	0.0006	0.0099	0.0074	0.0069	0.0188	0.0081	0.0149	0.0154	0.0084
Durables	0.0002	0.0601	-0.0152	0.0090	-0.0284	0.0014	-0.0837	0.0238	0.0512
	0.0007	0.0127	0.0095	0.0089	0.0241	0.0104	0.0191	0.0197	0.0108
M.Goods/S.	0.0084	0.0934	0.0177	0.0446	0.0460	0.0298	0.0044	-0.2145	0.2005
	0.0019	0.0337	0.0253	0.0236	0.0642	0.0277	0.0509	0.0525	0.0289
Savings	-0.0155	-0.1013	0.0694	0.0370	-0.0839	-0.0033	0.0949	0.0401	0.6075
	0.0032	0.0552	0.0414	0.0386	0.1051	0.0453	0.0833	0.0860	0.0473

	Expected prices										
	p_1	p_2	p_3	p_4	p_5	p_6	p_7	Yield	R^2	DW	DW4
Food	-0.0254	0.0191	-0.0007	-0.0288	0.0176	0.0006	-0.0091	-0.0025	0.53	1.69	2.45
	0.0163	0.0094	0.0096	0.0221	0.0122	0.0205	0.0211	0.0030			
Drink/Tob.	-0.0058	0.0039	0.0092	-0.0020	0.0106	-0.0098	-0.0148	-0.0026	0.67	1.76	2.29
	0.0147	0.0085	0.0087	0.0200	0.0110	0.0185	0.0191	0.0027			
Housing	-0.0110	-0.0003	0.0323	0.0766	0.0160	-0.0214	-0.0474	-0.0008	0.60	1.10	2.52
	0.0142	0.0082	0.0084	0.0194	0.0107	0.0180	0.0185	0.0026			
Clothing	0.0053	-0.0050	0.0033	-0.0076	0.0229	0.0165	-0.0449	0.0013	0.46	1.98	2.50
	0.0112	0.0064	0.0066	0.0152	0.0084	0.0141	0.0145	0.0020			
Fuel/Light	0.0104	0.0019	-0.0148	0.0236	0.0068	-0.0349	0.0088	0.0036	0.07	2.02	2.50
	0.0118	0.0068	0.0070	0.0161	0.0089	0.0149	0.0154	0.0022			
Durables	-0.0089	-0.0213	0.0064	0.0178	0.0137	0.0441	-0.0394	-0.0062	0.56	1.28	2.75
	0.0152	0.0088	0.0090	0.0207	0.0114	0.0192	0.0197	0.0028			
M.Goods/S.	-0.0517	-0.0416	0.0208	-0.0367	0.0228	0.0965	-0.0856	-0.0390	0.66	1.56	2.61
	0.0405	0.0233	0.0240	0.0551	0.0304	0.0511	0.0526	0.0074			
Savings	0.0871	0.0433	-0.0564	-0.0429	-0.1103	-0.0917	0.2324	0.0462	0.72	1.84	2.60
	0.0663	0.0382	0.0393	0.0901	0.0498	0.0836	0.0861	0.0120			

p_1 to p_7 are the prices in the same order as the equations. "Inc." denotes disposable income. The expectations on the 7 prices are the prices lagged three periods. "Yield" denotes gross flat yield on 2.5% consols, lagged one period. $\bar{R}^2$ denotes the coefficient of determination, corrected for degrees of freedom. DW is the Durbin-Watson statistic. DW4 is the Durbin-Watson statistic for fourth-order autocorrelation. 106 observations and 119 free coefficients. The loglikelihood is 3406.462. The figures show the coefficient estimates and the estimated asymptotic standard errors.

TABLE 5: THE MODEL UNDER ADDING-UP AND SYMMETRY

	Cons.	p_1	p_2	p_3	p_4	p_5	p_6	p_7	Inc.
Food	-0.0001	-0.0747	0.0122	0.0009	-0.0046	-0.0104	0.0390	0.0333	0.0543
	0.0008	0.0122	0.0074	0.0071	0.0082	0.0066	0.0088	0.0112	0.0118
Drink/Tob.	0.0013	0.0122	-0.0678	-0.0006	0.0076	-0.0050	-0.0088	0.0434	0.0410
	0.0007	0.0074	0.0087	0.0059	0.0062	0.0057	0.0074	0.0094	0.0102
Housing	0.0045	0.0009	-0.0006	-0.0831	-0.0024	-0.0053	0.0066	0.0339	-0.0023
	0.0007	0.0071	0.0059	0.0079	0.0058	0.0052	0.0065	0.0089	0.0099
Clothing	0.0011	-0.0046	0.0076	-0.0024	-0.0250	0.0009	-0.0037	0.0192	0.0570
	0.0005	0.0082	0.0062	0.0058	0.0159	0.0064	0.0104	0.0113	0.0078
Fuel/Light	0.0004	-0.0104	-0.0050	-0.0053	0.0009	-0.0041	-0.0021	0.0067	0.0042
	0.0006	0.0066	0.0057	0.0052	0.0064	0.0069	0.0066	0.0086	0.0083
Durables	0.0002	0.0390	-0.0088	0.0066	-0.0037	-0.0021	-0.0843	0.0268	0.0498
	0.0007	0.0088	0.0074	0.0065	0.0104	0.0066	0.0144	0.0126	0.0106
M.Goods/S.	0.0086	0.0333	0.0434	0.0339	0.0192	0.0067	0.0268	-0.1923	0.2013
	0.0019	0.0112	0.0094	0.0089	0.0113	0.0086	0.0126	0.0240	0.0279
Savings	-0.0159	0.0043	0.0190	0.0499	0.0080	0.0192	0.0263	0.0289	0.5947
	0.0032	0.0139	0.0114	0.0111	0.0134	0.0099	0.0133	0.0305	0.0462

	Expected prices							
	p_1	p_2	p_3	p_4	p_5	p_6	p_7	Yield
Food	-0.0103	0.0094	-0.0016	-0.0207	-0.0033	0.0040	0.0075	-0.0018
	0.0135	0.0095	0.0094	0.0211	0.0106	0.0201	0.0209	0.0030
Drink/Tob.	0.0069	0.0032	0.0040	0.0096	0.0088	-0.0152	-0.0159	-0.0012
	0.0110	0.0083	0.0079	0.0178	0.0092	0.0169	0.0181	0.0026
Housing	-0.0002	0.0025	0.0337	0.0858	0.0136	-0.0350	-0.0440	0.0008
	0.0109	0.0079	0.0081	0.0172	0.0087	0.0164	0.0177	0.0025
Clothing	0.0086	-0.0069	0.0007	-0.0055	0.0196	0.0182	-0.0456	0.0007
	0.0104	0.0063	0.0063	0.0148	0.0082	0.0135	0.0141	0.0020
Fuel/Light	0.0126	0.0075	-0.0102	0.0294	0.0094	-0.0502	0.0137	0.0044
	0.0092	0.0067	0.0065	0.0146	0.0074	0.0140	0.0150	0.0021
Durables	-0.0105	-0.0206	0.0055	0.0175	0.0201	0.0356	-0.0356	-0.0061
	0.0121	0.0085	0.0085	0.0189	0.0097	0.0178	0.0189	0.0027
M.Goods/S.	-0.0156	-0.0407	0.0180	0.0043	0.0146	0.0451	-0.0509	-0.0352
	0.0280	0.0223	0.0210	0.0489	0.0240	0.0457	0.0487	0.0070
Savings	0.0085	0.0455	-0.0501	-0.1203	-0.0829	-0.0025	0.1709	0.0383
	0.0456	0.0368	0.0345	0.0816	0.0394	0.0752	0.0806	0.0117

p_1 to p_7 are the prices in the same order as the equations. "Inc." denotes disposable income. The expectations on the 7 prices are the prices lagged three periods. "Yield" denotes gross flat yield on 2.5% consols, lagged one period. 106 observations and 98 free coefficients. The loglikelihood is 3384.106. The figures show the coefficient estimates and the estimated asymptotic standard errors.

The likelihood ratio test statistic for symmetry is 44.7. This is compared with a critical value of 38.9 for the Chi-square distribution with 21 degrees of freedom (1% significance level). Unfortunately no rigorously based small-sample correction is available, since the statistic (22) is only valid for the omitted variables case. Italianer (1985) nevertheless proposes a correction factor which is heuristically defined, by analogy with M/N in (22), as $\frac{1}{2}(pN - n_1)/pN + \frac{1}{2}(pN - n_2)/pN$, where n_1 and n_2 denote the numbers of unconstrained parameters under the alternative and the null, respectively. In our case we have $p = 7$, $N = 106$, $n_1 = 147$, and $n_2 = 126$, yielding a corrected test statistic of 36.49, which is no longer significant at the 1% level. We may then plausibly assume that the data do not allow us to reject symmetry. The largest eigenvalue of the Slutsky matrix was .0004; all the other eigenvalues were negative.

To conclude this section, we briefly compare our model and results with the existing literature on static demand systems. Probably the most popular such systems are the Rotterdam model (Barten, 1969) and the AIDS model (Deaton and Muellbauer, 1980). The symmetry and homogeneity restrictions have been tested, and strongly rejected, in both studies as well as in many others. A number of explanations (including small-sample bias) have been proposed for this phenomenon. One possibility, mentioned by Deaton and Muellbauer (1980, p.320), is a dynamic misspecification of some of the equations, due to the omission of conditioning variables or of price expectations. It is therefore of interest to estimate an ordinary, static, Rotterdam model with our data and compare the results with Table 4. The static Rotterdam model differs from Equation (19) in two respects: first, the expectations variables do not appear; second, the individual expenditures are explained by total expenditure, rather than by disposable income (since there is no equation for savings).

When symmetry is tested against adding-up in the static Rotterdam model, a likelihood ratio test statistic of 60.32 is obtained. The corrected statistic is in this case equal to $0.899 \times 60.32 = 54.2$, and considerably larger than the critical value of $\chi^2_{21;0.01} = 38.9$ which also applies to this case. Furthermore, the static Rotterdam model residuals exhibit higher first-order autocorrelation, with Durbin-Watson statistics of 1.43, 1.38, .80, 1.54, 1.81, 1.25 and 1.42 for the seven equations (instead of 1.69, 1.76, 1.10, 1.98, 2.02, 1.28, and 1.56 for the corresponding equations in Table 4). This can be viewed as evidence that the Deaton-Muellbauer conjecture is correct: the omission of price expectations in the static model leads to autocorrelated disturbances, which may lead to the improper rejection of symmetry.

Section 5. The model under strong intertemporal separability (SIS)

We now assume that the temporary utility functions satisfy:

$$u_t(x_t, X_{t+1}) = u_t^1(x_t) + u_t^2(X_{t+1})$$

so that the direct-indirect utility function (5) and its expectation (6) are also strongly separable. Under our assumption of myopia, which implies $\rho'_{t+1}a_{t+1} + I^*_{t+2} = r'_{t+1}a_{t+1}$ and $P'_{t+1} = (p'_{t+1}0\ldots0)$, we may then write (5) and (6) as:

$$v_t(x_t, P_{t+1}, w_{t+1} + \rho'_{t+1}a_{t+1} + I^*_{t+2}) = u_t^1(x_t) + v_t^2(p_{t+1}, w_{t+1} + r'_{t+1}a_{t+1})$$

$$F_t(x_t, a_{t+1}, \pi_t, A^*_{t+2}) = u_t^1(x_t) + F_t^2(a_{t+1} - a_t, \pi_t) \equiv F(y_t, \pi_t, t). \tag{23}$$

It will be recalled that $F(y_t, \pi_t, t)$ is maximized subject to $v'_t y_t = w_t + r'_t a_t$ with $v'_t = (p'_t \quad \imath'_m)$, yielding demand functions of the form (8). The classical results of comparative statics apply to the matrix Y_π of expectations derivatives (Phlips, 1974, p.183, eqs. 7.28 and 7.29), yielding:

$$Y_\pi = -(F_{yy}^{-1} - \Lambda_w^{-1} Y_w Y'_w) F_{y\pi} \tag{24}$$

where, as before, subscripts denote matrices of partial derivatives: F_{yy} has elements $\partial^2 F/\partial y_i \partial y_j$, $F_{y\pi}$ has elements $\partial^2 F/\partial y_i \partial \pi_k$, and Λ_w is the derivative with respect to $w_t + r'_t a_t$ of the Lagrange multiplier associated with the temporary budget constraint, $\lambda_t = \Lambda(p_t, w_t + r'_t a_t, \pi_t, t)$. Using (23), we may write:

$$F_{yy} = \begin{pmatrix} F_{xx} & O_{n\times m} \\ O_{m\times n} & F_{aa} \end{pmatrix} \qquad \text{and} \qquad F_{y\pi} = \begin{pmatrix} O_{n\times s} \\ F_{a\pi} \end{pmatrix}. \tag{25}$$

Combining (24) and (25), we see that the n first rows of Y_π and of S^* in (19) are collinear under SIS. On the other hand, since the substitution matrix K in Equation (11) consists of the first n columns of $\lambda_t(F_{yy}^{-1} - \Lambda_w^{-1} Y_w Y'_w)$ (see the Appendix), the m last rows of K and of S in (19) are collinear when (25) holds. So SIS introduces an asymmetry between the commodity equations and the asset equations in (19). Accordingly, we will now partition the rows of S, S^* and b into two groups of n rows and m rows, respectively. First, as we have just noted, (24), (25) and the definition of K imply:

$$K = \begin{pmatrix} K_{xx} \\ -\lambda_t \Lambda_w^{-1} Y_{w2} Y'_{w1} \end{pmatrix} \quad \text{and} \quad Y_\pi = \begin{pmatrix} O_{n\times s} \\ F_{aa}^{-1} F_{a\pi} \end{pmatrix} + Y_w(\Lambda_w^{-1} Y'_{w2} F_{a\pi}) \tag{26}$$

where Y_{w1} consists of the first n elements of Y_w, Y_{w2} consists of the last m elements of Y_w, and where K_{xx} is the upper $n \times n$ block of K. Since the last "prices" are unity, we have under adding-up that:

$$p'_t K_{xx} - \imath'_m \lambda_t \Lambda_w^{-1} Y_{w2} Y'_{w1} = p'_t K_{xx} - \lambda_t \Lambda_w^{-1} (\imath'_m Y_{w2}) Y'_{w1} = 0,$$

and from the symmetry of K_{xx}, it follows that:

$$K_{xx} p_t - \lambda_t \Lambda_w^{-1} (\imath'_m Y_{w2}) Y_{w1} = 0. \tag{27}$$

We now partition the vector b in (19) as $b = (b_1' \quad b_2')'$, where b_1 contains the n propensities to spend and where b_2 contains the m propensities to invest. Note that $b_2 = Y_{w2}$ since the last "prices" are unity. Equation (27) may then be rewritten as:

$$b_1 = \beta S_{xx} \imath_n \quad \text{with } \beta = \mu_t \lambda_t^{-1} \Lambda_w (\imath_m' b_2)^{-1} \tag{28}$$

where it will be recalled that $\mu_t = v_t' y_t$ and where S_{xx} is the upper $n \times n$ block of S. In other words, in each commodity equation, the marginal propensity to spend must be proportional to the sum of the price coefficients.

It is easily shown that β^{-1} can be interpreted as a partial income flexibility (Theil, 1975, p.29), and should be negative. Indeed, since the null-space of $\lambda_t(F_{yy}^{-1} - \Lambda_w^{-1} Y_w Y_w')$ is spanned by v_t (Barten, Kloek and Lempers (1969)) and since $v_t' Y_w = 1$, we have:

$$\lambda_t (p_t' F_{xx}^{-1} \quad \imath_m' F_{aa}^{-1}) = \lambda_t \Lambda_w^{-1} Y_w', \qquad \text{so that:}$$

$$\lambda_t \imath_m' F_{aa}^{-1} \imath_m = \lambda_t \Lambda_w^{-1} Y_{w2}' \imath_m = \mu_t \beta^{-1}.$$

The negativity of β^{-1} follows from the negative definiteness of F_{aa} and from $\lambda_t > 0$, $\mu_t > 0$.

Upon using the definitions of the parameter matrices a, b, S, S^* in (19), upon taking into account the adding-up and symmetry restrictions, and upon incorporating the SIS restrictions implied by (24) to (28), we obtain the following constraints on the parameters of (19):

$$\imath_{n+m}' a = 0 \tag{29}$$

$$S = \begin{pmatrix} S_{xx} \\ -\phi b_2 b_1' \end{pmatrix}, \quad S_{xx} = S_{xx}', \quad \imath_{n+m}' S = O_{1\times n} \tag{30}$$

$$b = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}, \quad b_1 = \beta S_{xx} \imath_n, \quad \imath_{n+m}' b = 1 \tag{31}$$

$$S^* = \begin{pmatrix} O_{n\times s} \\ S_a \end{pmatrix} + bk', \quad \imath_m' S_a = -k' \tag{32}$$

where $\phi = \lambda_t \Lambda_w^{-1} \mu_t^{-1}$, $\beta = \phi^{-1}(\imath_m' b_2)^{-1}$, $S_a = -F_{aa}^{-1} F_{a*} \mu_t^{-1}$, and $k' = \Lambda_w^{-1} b_2' F_{a*} \mu_t^{-1}$.

We will now reparameterize Equation (19), and show that the new model is quadrilinear, with linear constraints on the new parameters. This will provide the key to estimating Equation (19) under SIS. It will be convenient to define $\gamma = (\imath_m' b_2)^{-1} b_2$, a normalized vector of marginal propensities to invest. From (31) and from $\phi = \beta^{-1}(\imath_m' b_2)^{-1}$, we then have $\phi b_2 b_1' = \gamma \imath_n' S_{xx}$. Next, from (31) and the definition of γ, we have:

$$b_2 = (\imath_m' b_2)\gamma = (1 - \imath_n' b_1)\gamma = (1 - \beta \imath_n' S_{xx} \imath_n)\gamma.$$

Substituting these equalities into (30)-(32), we obtain:

$$S = \begin{pmatrix} I_n \\ -\gamma \imath_n' \end{pmatrix} S_{xx} \tag{33}$$

$$b = \begin{pmatrix} \beta S_{xx} \imath_n \\ (1 - \beta \imath_n' S_{xx} \imath_n)\gamma \end{pmatrix} \tag{34}$$

$$S^* = \begin{pmatrix} O_{n\times s} \\ S_a \end{pmatrix} + \begin{pmatrix} \beta S_{xx} \imath_n \\ (1 - \beta \imath_n' S_{xx} \imath_n)\gamma \end{pmatrix} k'. \tag{35}$$

From Equations (19) and (33)–(35), we see that the parameters of the new model are the elements of a, S_{xx}, γ, β, S_a, and k. Adding-up implies the linear restrictions $\imath_{n+m}'a = 0$, $\imath_m'\gamma = 1$, and $\imath_m' S_a = -k'$. These constraints are conveniently imposed by deleting any asset demand equation. The symmetry restriction $S_{xx} = S_{xx}'$ can be imposed by a further reparameterization in the lower triangle of S_{xx} (see Deschamps (1988)). Quadrilinearity follows from Equation (35), where it is seen that S^* involves products of four different types of primitive parameters (in β, S_{xx}, γ, and k).

Section 6. Maximum likelihood estimation under SIS

Straightforward, but complicated, matrix algebra shows that four linear parameterizations can be obtained from (19) when S, b, and S^* are given by (33)–(35). Given the remaining parameters, the first one is linear in $(a \quad \gamma \quad S_a)$; the second one in S_{xx}; the third one in k; the last one in β. The technical details will be sent to any interested reader; the noteworthy feature is that quadrilinearity greatly facilitates the analytical derivation of the score vector and of the information matrix, which are necessary in an application of the method of scoring (see, e.g., Quandt (1983, p.718); and for the very similar problem of estimating the linear expenditure system, Parks (1971)). In our context, the method of scoring was found to have convergence properties superior to other, more widely used methods, such as the Davidon-Fletcher-Powell algorithm.

One difficulty encountered in the estimation of the model under SIS should be mentioned, however. This is the lack of local identification: it is clear from (34) and (35) that $S_{xx}\imath_n \neq O_{n\times 1}$ is necessary for the identification of both β and k. This may cause numerical difficulties in an application of the method of scoring, since the information matrix can become ill-conditioned when $S_{xx}\imath_n$ is close to the null vector. In this case the inverse information matrix will fail to provide a good approximation of the inverse Hessian and convergence will be very slow. When (and only when) this happens, faster convergence can often be obtained with the Davidon-Fletcher-Powell algorithm, initially using the inverse information matrix in the updating formula.

Table 6 presents the results for the model under SIS. The estimate of β, -2.018 with an estimated standard error of .6223, is indeed significantly negative. The asymptotic standard errors for b and S^* in (34) and (35) were estimated by the usual approximation:

$$\widehat{\mathrm{Var}}\left(g(\hat{\theta})\right) = \left(\nabla g(\hat{\theta})\right) \hat{\Sigma} \left(\nabla g(\hat{\theta})\right)',$$

where $\hat{\Sigma}$ is the estimated asymptotic covariance matrix of $\hat{\theta}$ and $\nabla g(\hat{\theta})$ is the matrix of the first derivatives of g (Monfort (1980, p.166)).[5]

TABLE 6: THE MODEL UNDER ADDING-UP, SYMMETRY AND SIS

	Cons.	p_1	p_2	p_3	p_4	p_5	p_6	p_7	Inc.
Food	0.0002	-0.0871	0.0137	0.0118	-0.0011	0.0007	0.0374	0.0070	0.0355
	0.0005	0.0093	0.0059	0.0057	0.0062	0.0055	0.0068	0.0095	0.0078
Drink/Tob.	0.0015	0.0137	-0.0640	0.0051	0.0088	-0.0015	-0.0078	0.0271	0.0379
	0.0004	0.0059	0.0073	0.0051	0.0049	0.0051	0.0058	0.0081	0.0067
Housing	0.0039	0.0118	0.0051	-0.0706	-0.0070	-0.0021	0.0050	0.0425	0.0308
	0.0004	0.0057	0.0051	0.0069	0.0046	0.0046	0.0052	0.0078	0.0068
Clothing	0.0015	-0.0011	0.0088	-0.0070	-0.0322	-0.0005	-0.0020	0.0095	0.0493
	0.0004	0.0062	0.0049	0.0046	0.0101	0.0051	0.0079	0.0090	0.0065
Fuel/Light	0.0005	0.0007	-0.0015	-0.0021	-0.0005	-0.0012	0.0017	-0.0006	0.0071
	0.0003	0.0055	0.0051	0.0046	0.0051	0.0065	0.0056	0.0079	0.0051
Durables	-0.0003	0.0374	-0.0078	0.0050	-0.0020	0.0017	-0.0892	0.0284	0.0533
	0.0005	0.0068	0.0058	0.0052	0.0079	0.0056	0.0116	0.0110	0.0079
M.Goods/S.	0.0089	0.0070	0.0271	0.0425	0.0095	-0.0006	0.0284	-0.1974	0.1684
	0.0015	0.0095	0.0081	0.0078	0.0090	0.0079	0.0110	0.0221	0.0231
Savings	-0.0162	0.0176	0.0188	0.0153	0.0244	0.0035	0.0264	0.0834	0.6177
	0.0029	0.0055	0.0054	0.0048	0.0069	0.0027	0.0072	0.0215	0.0426

Expected prices

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	Yield
Food	0.0033	-0.0040	0.0031	0.0146	0.0110	0.0004	-0.0242	-0.0004
	0.0038	0.0030	0.0028	0.0071	0.0040	0.0060	0.0080	0.0009
Drink/Tob.	0.0036	-0.0042	0.0033	0.0156	0.0118	0.0004	-0.0257	-0.0004
	0.0040	0.0031	0.0030	0.0072	0.0039	0.0064	0.0079	0.0010
Housing	0.0029	-0.0035	0.0027	0.0127	0.0096	0.0003	-0.0210	-0.0004
	0.0033	0.0026	0.0025	0.0061	0.0033	0.0052	0.0068	0.0008
Clothing	0.0046	-0.0055	0.0043	0.0203	0.0153	0.0005	-0.0335	-0.0006
	0.0053	0.0041	0.0038	0.0093	0.0044	0.0084	0.0092	0.0013
Fuel/Light	0.0007	-0.0008	0.0006	0.0029	0.0022	0.0001	-0.0048	-0.0001
	0.0009	0.0008	0.0007	0.0025	0.0017	0.0012	0.0036	0.0002
Durables	0.0050	-0.0060	0.0047	0.0219	0.0166	0.0005	-0.0363	-0.0006
	0.0057	0.0044	0.0041	0.0100	0.0051	0.0090	0.0104	0.0014
M.Goods/S.	0.0158	-0.0188	0.0148	0.0692	0.0523	0.0017	-0.1145	-0.0020
	0.0178	0.0139	0.0131	0.0310	0.0164	0.0285	0.0322	0.0044
Savings	-0.0359	0.0428	-0.0336	-0.1572	-0.1188	-0.0038	0.2599	0.0044
	0.0405	0.0313	0.0296	0.0698	0.0354	0.0648	0.0697	0.0099

p_1 to p_7 are the prices in the same order as the equations. "Inc." denotes disposable income. The expectations on the 7 prices are the prices lagged three periods. "Yield" denotes gross flat yield on 2.5% consols, lagged one period. 106 observations and 44 free coefficients. The loglikelihood is 3318.409. The figures show the coefficient estimates and the estimated asymptotic standard errors.

As before, the equation for fuel and light shows little significance. The price coefficients in the equation for savings are significant except for fuel and light. The significance of the expected price coefficients in that equation is not markedly different from the one observed under symmetry; things change dramatically, however, when one considers the interest rate variable. The eigenvalues of the Slutsky matrix were all negative in this case.

The test statistic for symmetry and SIS against symmetry only is 131.4; the 1% critical value of the Chi-square distribution with $98 - 44 = 54$ degrees of freedom is 81.07. With 106 observations, it is unlikely that this strong significance is due to small-sample bias, and we conclude that SIS is strongly rejected. This is hardly surprising: the SIS restriction more than halves the number of free parameters in the unconstrained demand system. Even though this result was obtained under the maintained restriction of myopia, there is no obvious reason for the conclusion to be different in the (possibly intractable) fully intertemporal model.

Section 7. Conclusions

This paper has formulated a fairly general model of consumption and investment, and given a detailed discussion of the restrictions needed for its empirical viability. The model was satisfactorily estimated with quarterly British data on seven consumption goods and a single composite asset, defined as savings. It was argued that the new model provides some empirical improvement over a static system of demand equations, in the sense that it decreases autocorrelation and lowers the test statistic for symmetry. Being based on the differential system approach, it is also more flexible than the existing empirical models of portfolio selection.

The assumption of strong intertemporal separability was formulated as a nested hypothesis, and strongly rejected by a likelihood ratio test. In the light of the ubiquitous presence of SIS in the empirical literature, we view this as the most significant contribution of this paper.

Left to further research is the estimation of the model with several assets. Their list should be reasonably exhaustive, so that wealth can provide an accurate estimation of disposable income. The observations should also span a sufficiently long period. The collection of such a sample, however, is a major research project in its own right.

Also left to further research is the estimation of the new model with a richer specification of expectations that might include second-order moments. In the context of our model, this would merely add to the list of explanatory variables, and to the number of columns in our matrix S^*. In view of Equation (34), it would therefore increase the number of restrictions under SIS. For this reason, the rejection of SIS appears likely in the richer models as well as in the present one.

APPENDIX: DERIVATION OF THE SLUTSKY EQUATIONS

The first-order conditions for Problem (P') in Section 3 are:

$$F_y(y_t, \pi_t, t) = \lambda_t v_t \tag{36}$$

$$v_t' y_t = w_t + r_t' a_t \tag{37}$$

$$\pi_t = \Pi(p_t, w_t, r_t, \gamma_t, t) \tag{38}$$

where $y_t' = (x_t' \quad (a_{t+1} - a_t)')$ and $v_t' = (p_t' \quad \iota_m')$. The solution of (36) to (38) is:

$$y_t = \bar{Y}(p_t, w_t, r_t, a_t, \gamma_t, t) \tag{39}$$

$$\lambda_t = \bar{\Lambda}(p_t, w_t, r_t, a_t, \gamma_t, t). \tag{40}$$

If we denote by ℓ the dimension of γ_t, the total differentials of (36) and (37) may be written as:

$$\begin{pmatrix} F_{yy} & v_t & F_{y\pi} & F_{yt} \\ v_t' & 0 & O_{1\times s} & 0 \end{pmatrix} \begin{pmatrix} dy_t \\ -d\lambda_t \\ d\pi_t \\ dt \end{pmatrix} =$$

$$\begin{pmatrix} \lambda_t I_{n+m} & O_{(n+m)\times 1} & O_{(n+m)\times m} & O_{(n+m)\times m} & O_{(n+m)\times \ell} & O_{(n+m)\times 1} \\ -y_t' & 1 & a_t' & r_t' & O_{1\times \ell} & 0 \end{pmatrix} \begin{pmatrix} dv_t \\ dw_t \\ dr_t \\ da_t \\ d\gamma_t \\ dt \end{pmatrix} \tag{41}$$

whereas Equations (38), (39) and (40) imply:

$$\begin{pmatrix} dy_t \\ -d\lambda_t \\ d\pi_t \\ dt \end{pmatrix} = \begin{pmatrix} \bar{Y}_p & \bar{Y}_w & \bar{Y}_r & \bar{Y}_a & \bar{Y}_\gamma & \bar{Y}_t \\ -\bar{\Lambda}_p & -\bar{\Lambda}_w & -\bar{\Lambda}_r & -\bar{\Lambda}_a & -\bar{\Lambda}_\gamma & -\bar{\Lambda}_t \\ \Pi_p & \Pi_w & \Pi_r & O_{s\times m} & \Pi_\gamma & \Pi_t \\ O_{1\times n} & 0 & O_{1\times m} & O_{1\times m} & O_{1\times \ell} & 1 \end{pmatrix} \begin{pmatrix} dp_t \\ dw_t \\ dr_t \\ da_t \\ d\gamma_t \\ dt \end{pmatrix}. \tag{42}$$

We note that the definition of v_t implies:

$$dv_t = \begin{pmatrix} dp_t \\ O_{m\times 1} \end{pmatrix} = J dp_t, \quad \text{with } J = \begin{pmatrix} I_n \\ O_{m\times n} \end{pmatrix}$$

and also note that $y_t' J = x_t'$ from the definition of y_t. We may then combine (41) and (42) as:

$$\begin{pmatrix} F_{yy} & v_t & F_{y\pi} & F_{yt} \\ v_t' & 0 & O_{1\times s} & 0 \end{pmatrix} \begin{pmatrix} \bar{Y}_p & \bar{Y}_w & \bar{Y}_r & \bar{Y}_a & \bar{Y}_\gamma & \bar{Y}_t \\ -\bar{\Lambda}_p & -\bar{\Lambda}_w & -\bar{\Lambda}_r & -\bar{\Lambda}_a & -\bar{\Lambda}_\gamma & -\bar{\Lambda}_t \\ \Pi_p & \Pi_w & \Pi_r & O_{s\times m} & \Pi_\gamma & \Pi_t \\ O_{1\times n} & 0 & O_{1\times m} & O_{1\times m} & O_{1\times \ell} & 1 \end{pmatrix} =$$

$$\begin{pmatrix} \lambda_t J & O_{(n+m)\times 1} & O_{(n+m)\times m} & O_{(n+m)\times m} & O_{(n+m)\times \ell} & O_{(n+m)\times 1} \\ -x_t' & 1 & a_t' & r_t' & O_{1\times \ell} & 0 \end{pmatrix}.$$

The last equality implies the fundamental matrix equation:

$$\begin{pmatrix} F_{yy} & v_t \\ v_t' & 0 \end{pmatrix} \begin{pmatrix} \bar{Y}_p & \bar{Y}_w & \bar{Y}_r & \bar{Y}_a & \bar{Y}_\gamma & \bar{Y}_t \\ -\bar{\Lambda}_p & -\bar{\Lambda}_w & -\bar{\Lambda}_r & -\bar{\Lambda}_a & -\bar{\Lambda}_\gamma & -\bar{\Lambda}_t \end{pmatrix} =$$

$$\begin{pmatrix} \lambda_t J - F_{y\pi}\Pi_p & -F_{y\pi}\Pi_w & -F_{y\pi}\Pi_r & O_{(n+m)\times m} & -F_{y\pi}\Pi_\gamma & -F_{y\pi}\Pi_t - F_{yt} \\ -x_t' & 1 & a_t' & r_t' & O_{1\times \ell} & 0 \end{pmatrix}. \tag{43}$$

The Slutsky equations (11) to (16) are the solution of (43) for the derivatives of $\bar{Y}$. Upon letting:

$$\begin{pmatrix} F_{yy} & v_t \\ v_t' & 0 \end{pmatrix}^{-1} = \begin{pmatrix} F^{yy} & z \\ z' & d \end{pmatrix},$$

we have in particular:

$$\bar{Y}_p = F^{yy}(\lambda_t J - F_{y\pi}\Pi_p) - zx_t'$$

which is identical to (11) with $K = \lambda_t F^{yy} J$, $Y_w = z$ and $Y_\pi = -F^{yy}F_{y\pi}$. This substantiates our claim that the upper $n \times n$ block of K is symmetric and negative definite. Furthermore, from the partitioned inversion formula, we have that $F^{yy} = F_{yy}^{-1} + d^{-1}zz'$; Equation (24) then follows from $Y_\pi = -F^{yy}F_{y\pi}$, $z = Y_w$ and $d = -\Lambda_w$ (see Phlips (1974, pp. 182–183)).

FOOTNOTES

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2. Another possibility is to treat π_t as a vector of observable predictors. Unfortunately, such data are not generally available.

3. For additional details on the form of (19) that can be used for estimation, see Barten and Geyskens (1975).

4. In the sequel, we denote a null matrix with α rows and β columns as $O_{\alpha\times\beta}$, and an identity matrix of order α as I_α.

5. Note that the lower block of S in (33) reduces to $-\iota_n' S_{xx}$, since the model was estimated with a single asset.

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